A Hybrid Finite Element Method for Axisymmetric Waveguide fed Horns *

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Abstract

A new method for finding radiation patterns and the reflection coeffi-
cients associated with an axisymmetric waveguide fed horn is presented.
The approach is based on a hybrid finite element method (FEM) wherein
the electromagnetic fields in the FEM region are coupled to the fields
outside by two surface integral equations. Because of the local nature of
the FEM, this formalism allows for the presence of inhomogeneities to be
included in the problem domain. The matrix equation which results from
the application of this method is shown to be complex-symmetric. It is,
furthermore, diagonally dominant and sparse. Comparisons of calculated
and measured data for two different horns show good agreement.

I. INTRODUCTION

The problem of finding radiation patterns and the reflection coefficients of an
axisymmetric waveguide fed horn is clearly important to designers as this kind
of horn is used both as an antenna and as a feed. Although design principles for
many of the specific horns of this type are well known- for instance, a corrugated
horn [1,2], dielectric rod [3], and dielectric horn [4]- a tool with more accurate
predictive powers is often needed. This is especially true for those horns which
use a combination of design rules such as the feed horn shown in Fig. 1. This
horn features a flange section, a dielectric surface-wave element, and is covered
by a radome.

in the absence of inhomogeneities, an approach which can find the desi red
quantities was developed by Berthon and Bills [5] using the method of mo-
ments (MoM) in conjunction with a single waveguide mode serving as the sole
excitation for the fields. Their method, while yielding very accurate reflec-
tion coefficients and radiation patterns, does not apply to horns with inhomo-
geneities due to the fact that their integral equation kernels are derived from free

space Green's functions. As is well known, however, the finite element method (FEM) can address problems with inhomogeneities. Consequently, this paper will present a FEM formulation which solves the problem of radiation from axisymmetric waveguide fed horns with or without inhomogeneities in-escnt.

In this approach, the FEM is coupled to both the MoM and a modal technique via surface integrals. The FEM region contains all the inhomogeneities. The FEM/MoM boundary serves to couple the local FEM electromagnetic fields Lo the external radiated fields. This type of FEM/MoM coupling has been used previously for scattering problems [6,7] and it is on the particular approach given in [7], the Hybrid Symmetric Finite Element Method (HSFEM), that this present work is based. The FEM/modal boundary, on the other hand, provides a means for waveguide modes to excite the local FEM fields thus transforming the technique from one that solves scattering problems to one that addresses radiation problems. Following [7, 8], we describe, in section 111, this FEM/modal coupling. It is noted that the J';M/mods) hybridization has it's roots in the waveguide discontinuity problem [9, 10] wherein modes were used to reprc.sent the fields in the uniform waveguide sections while the FEM was used to model the locally anisotropic arid inhomogeneous regions.

Finally, In section IV, the technique is applied to the horns shown in Figs. 3 and 8. Comparisons with measured data arc made.

II. PROBLEM DESCRIPTION

Consider the generic horn in Fig. 1. As usual, the original three dimensional electromagnetic problem may be reduced to a series of uncoupled two dimensional problems by expanding out the azimuthal variable, ϕ, in a Fourier series. Fig. 2 then shows the computational domain which must be solved for each azimuthal harmonic of interest.

Clearly, the mesh in Fig. 2 has two disjoint boundaries, S_1 and S_2. Consider S_1. When revolved about the axis of rotation, the sur face generated by S_1 completely envelopes the mesh. It is on this boundary that an integral equation is applied to couple the fields within the finite element region to the external radiation fields. The other boundary, S_2, consists of two separate parts. One part Of this Surface, S_{2A}, is made perpendicular to the z-axis and extends from $\rho = 0$ to $\rho = a$. The surface of revolution generated by this line segment corresponds to a disk spanning the throat of the horn arid it is on this mesh boundary that the finite element region is coupled to waveguide modes. The remainder of the surface, S_{2B}, is made conformal Lo both the inside and outside walls of the horn. As a result, all metal edges which may contribute to diffraction arc included in the finite element mesh. Note that the S_{2A} surface is placed at $z = 0$. This can always be done as the geometry is translationally invariant in the z direction.

Our subsequent discussion will be concerned only with the FEM/modal coupling via the S_{2A} mesh boundary.

III. FORMULATION

The coupling of the waveguide modes to the finite element region is accomplished by two weak-form1 integral equations (IE). These two IE must be tested properly in order to generate a symmetric system matrix. Hence, in this section, we will state these two 1 E, show the appropriate test Vectors, then show and discuss the resultant system.

The first key weak-form IE may be obtained by conjugate testing the electric field wave equation with the set of finite element basis functions, denoted here by w,

$$\mathbf{w} \cdot \mathbf{e}^{FEM}, \qquad \text{for } i = 1, 2, \ldots N_{FEM}. \tag{1}$$

As in [7,9], these FEM basis functions are chosen such that first order node (edge) based vector functions are used to expand the ϕ (transverse to ϕ) directed field components. The resulting IE is

$$\int_V \frac{1}{\mu_r} \left(\mathbf{v} \times \mathbf{w}^* \right) \cdot \left(\mathbf{v} \times E \right) dV - \int_V k_0^2 \epsilon_r \mathbf{w}^* \cdot \mathbf{1} \cdot dV$$

$$j\omega\mu_0 \int_{S_1} w \cdot (\hat{n} \times H) dS_1$$

$$j\omega\mu_0 \int_{S_{2A}} \mathbf{w}^* (\hat{n} \times H) dS_{2A} = 0, \tag{2}$$

where the electric and magnetic fields, E and H, are unknowns. The relevant part of this equation is the S_{2A} surface integration term as this surface is identified with the FEM/waveguide interface. On this surface, we expand the magnetic field as series of waveguide modes (see Appendix),

$$H|_{S_{2A}} = H^{WG}|_{z=0}$$

$$= \sum_{p=1}^{N_{WG}} (a_p - b_p) \, h_p^{WG}$$

$$= \sum_{p=1}^{N_{WG}} (a_p - b_p) \frac{1}{Z_p^{WG}} \left(\mathbf{a}_z \times \mathbf{e}_p^{WG} \right), \tag{3}$$

which when substituted back into equation (2), clearly couples finite elements to waveguide modes. This coupling, however, is not yet complete as the preceeding substitution enforces only the tangential magnetic field continuity across S_{2A}. To complete the coupling, the continuity of the electric field across S_{2A} must be enforced:

$$E^{FEM}|_{z=0} = E^{WG}|_{z=0}. \tag{4}$$

When conjugate tested with another set of vectors, u, this equation gives the second key weak-form equation:

$$\int_{S_{2A}} u^* \cdot E^{FEM} dS_{2A} = \int_{S_{2A}} u^* \cdot E^{WG} dS_{2A}, \tag{5}$$

Finally, the right-hand side excitation vectors, a^B rrnd a^E, are given by

$$a^B(i) = \sum_{j=1}^{NWG} a_j B^{EB}(i,j), \tag{11}$$

and

$$a^E(i) = - a_i K^{BB}(i,i). \tag{12}$$

These vectors involve summations over the complete set of waveguide modes as they represent the excitaion of a single finite element by the total forward travelling field in the waveguide.

IV. NUMERICAL RESULTS

In this section, we will compare measured and simulated data for the two horns shown in Figs. 3 and 8.

The horn shown in Fig. 3 is a Low Gain Antenna (LGA) to be used aboard the NASA/1 1'1, Mars Pathfinder spacecraft L. It radiates dominant mode RCP and operates with transmit and receive frequencies at 7.175 and 8.425 GHz, respectively. The chokes are used to null out aperture currents at the two opm-sting frequencies thus insuring radiation in a balanced hybrid mode: small axial ratio and low cross polarization. Note the dielectric plug in the aperture anct filling the chokes. This is to prevent Martian dust from entering the horn. The relative permittivity of the dielectric material used is 1.07.

The corresponding two dimensional modelling domain, depicted in Fig. 4, clearly shows where the FEM/MoM and the FEM/modal boundary surfaces are located, This region was meshed using approximately 35 linear nodes per wavelength at the highest frequency of simulation, 8.6 GHz, and 8 waveguide modes of azimuthal index, $n = 1$, were allowed to exist at the FEM/modal interface. It is noted that this choice of mesh density and modal content yields a highly converged solution as a mesh of 25 nodes per wavelength and 4 waveguide modes yielded identical far field radiation patterns and reflection coefficients that differed by only 0.3 dB at the -20 dB level. A matrix of order N = 5073 results.

Fig. 5 shows the calculated and measured reflection coefficient over a band of frequencies extending from 7.0 GHz to 8.6 GHz. The measured coefficient was not time gated and thus includes the combined effects of a reflection due to the horn with a reflection due to a polarizer used to transition from rectangular to circular waveguide. The polarizer is, however, placed far enough behind the hack of the horn such that the evanescent higher order modes generated do not couple into and through the horn. Figs. 6 and 7 show the far field patterns at 7.175 GHz and 8.425 GHz. Good agreement may be stated.

The method is now applied to the wide band horn antenna shown in Fig. 8. This horn was designed by 1 'hilip St anton of the Jet Propulsion Laboratory to

operate from 5.5 - 10.0 GHz. It features corrugations to equalize the E- and H-plane patterns and a dielectric surface wave element for frequency independent beamwidth over the whole band. To obtain a good match, the corrugations were ring-loaded at the back-end of the horn, tapered through the nlid-section, then flanged at the aperture. The computational domain for this horn is shown in Fig. 9. Note that the FEM/modal boundary is placed just behind the back of the surface wave element inhomogeneity.

The horn was simulated once with LCP excitation ($n = -1$) then again with RCP excitation ($n = 1$). The results from these two runs were combined to generate the linear E- and H-plane patterns shown. For these simulations, the computational domain was meshed at a linear density of approximately 31 nodes per wavelength at the highest frequency of simulation thus giving rise to a system matrix of order $N = 37,427$. Again, it is noted that this high mesh density was chosen in order to guarantee that the solution obtained has already converged.

Fig. 10 shows the measured reflection coefficient over the whole band and the calculated value at a discreet set of frequencies. The measured coefficient was time gated to remove a reflection from the orthomode junction used to transition from coax to waveguide. Although this junction dots excite higher order modes, these evanescent modes do not couple into and through the horn as this discontinuity is located far behind the track of the horn. Fig. 11 shows the far field pattern at 8.2 GHz. Far field patterns at other frequencies over the band rdso show good agreement. For comparison, Fig. 12 shows the far field pattern of the horn without dielectric surface wave element at the same frequency, 8.2 GHz. Note that the E- and H-plane patterns are not coincidental when the surfrrce-wave element is absent.

APPENDIX

The fields in the waveguide are written as:

$$E^{WG} = \sum_{p=1}^{N_{WG}} a_p c_p e^{-jk_{s,p}z} + b_p c_p e^{+jk_{s,p}z} \tag{13}$$

$$H^{WG} = \sum_{p=1}^{N_{WG}} a_p h_p e^{-jk_{s,p}z} - b_p h_p e^{+jk_{s,p}z} \tag{14}$$

where a_p (b_p) are the expansion coefficients for the forward (backward) modes in the guide. The transverse mode vectors in collations (13) and (14) are given by

$$c_p^{TM} = j\sqrt{\frac{|k_{z,p}|}{\omega \epsilon \pi}} \frac{J_n'(\frac{x_{np}\rho}{a})}{aJ_{n+1}(x_{np})} e^{+jn\phi} a_\rho$$

$$+ \sqrt{\frac{|k_{z,p}|}{\omega \epsilon \pi}} \frac{n}{x_{np}} \frac{J_n(\frac{x_{np}\rho}{a})}{\rho J_{n+1}(x_{np})} e^{-jn\phi} \mathbf{a}_\phi \tag{15}$$

$$\mathbf{h}_p^{TM} = \frac{\omega \epsilon}{k_{z,p}} (\mathbf{a}_z \times \mathbf{e}_p^{TM}) = \frac{1}{Z_p^{TM}} (\mathbf{a}_z \times \mathbf{e}_p^{TM}) \tag{16}$$

$$\mathbf{e}_p^{TE} = +j \sqrt{\frac{\omega \mu}{|k_{z,p}|\pi}} \frac{n}{\sqrt{x_{np}'^2 - n^2}} \frac{J_n(\frac{x_{np}'\rho}{a})}{\rho J_n(x_{np}')} e^{-jn\phi} \mathbf{a}_\rho$$

$$+ \sqrt{\frac{\omega \mu}{|k_{z,p}|\pi}} \frac{x_{np}'}{\sqrt{x_{np}'^2 - n^2}} \frac{J_n'(\frac{x_{np}'\rho}{a})}{a J_n(x_{np}')} e^{-jn\phi} \mathbf{a}_\phi \tag{17}$$

$$\mathbf{h}_p^{TE} = \frac{k_{z,p}}{\omega \mu} (\mathbf{a}_z \times \mathbf{e}_p^{TE}) = \frac{1}{Z_p^{TE}} (\mathbf{a}_z \times \mathbf{e}_p^{TE}) \tag{18}$$

where a is the waveguide radius and

$$k_{z,p}^{TE} = k_0^2 - \frac{x_{np}^2}{a}$$

$$k_{z,p}^{TM} = k_0^2 - \frac{x_{np}'^2}{a}.$$

These vectors are nearly identical to the standard set given in Marcuvitz [1]] but differ in three important respects. First, these mode vectors are written as harmonics of the azimuthal angle, ϕ. Second, the choice of phase on the mode vectors is not arbitrary. in order to generate a symmetric system matrix (8), the $z(\rho)$ components of $\mathbf{e}_p^{WG}$ must be purely real (imaginary) for both propagating and evanescant modes. This point is explained in [7] and has to do with the choice of finite element basis function. q'bird, these mode vectors satisfy power ortho-normalization:

$$\int_S \mathbf{e}_m \times \mathbf{h}_n^* \, dS = \begin{cases} \delta_{mn}, & \text{for propagating modes} \\ -j\delta_{mn}, & \text{for evanescant TM modes} \\ j\delta_{mn}, & \text{for evanescant TE mocks} \end{cases}$$

This normalization is convenient for S parameter calculations.

ACKNOWLEDGEMENT

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Figure Titles

Fig. 1. Generic axisymmetric horn.

Fig. 2. Computational domain for horn shown in Figure 1.

Fig. 3. Pathfinder spacecraft low gain horn antenna (1 GA).

Fig. 4. Computational domain for Pathfinder I, CA.

Fig. 5. Measured and calculated reflection coefficient for Pathfinder LG A.

Fig. 6. Pathfinder LGA far field pattern at $f_0 = 7.175$ GHz.

Fig. 7. Pathfinder LGA far field pattern at $f = 8.425$ GHz.

Fig. 8. Prime focus feed horn (PFF) with coaxial surface wave element (SWE)

Fig. 9. Computational domain for PPF horn.

Fig.10. Measured and calculated reflection coefficient for PFF horn with SWE.

Fig. 11. Far field pattern for PFF horn with SWE at $f = 8.2$ GHz.

Fig. 12. Far field pattern for PFF horn without SWE at $f = 8.2$ GHz.

Figure 1. Generic horn with axial symmetry

dielectric radorne

coaxial dielectric rod

flange section
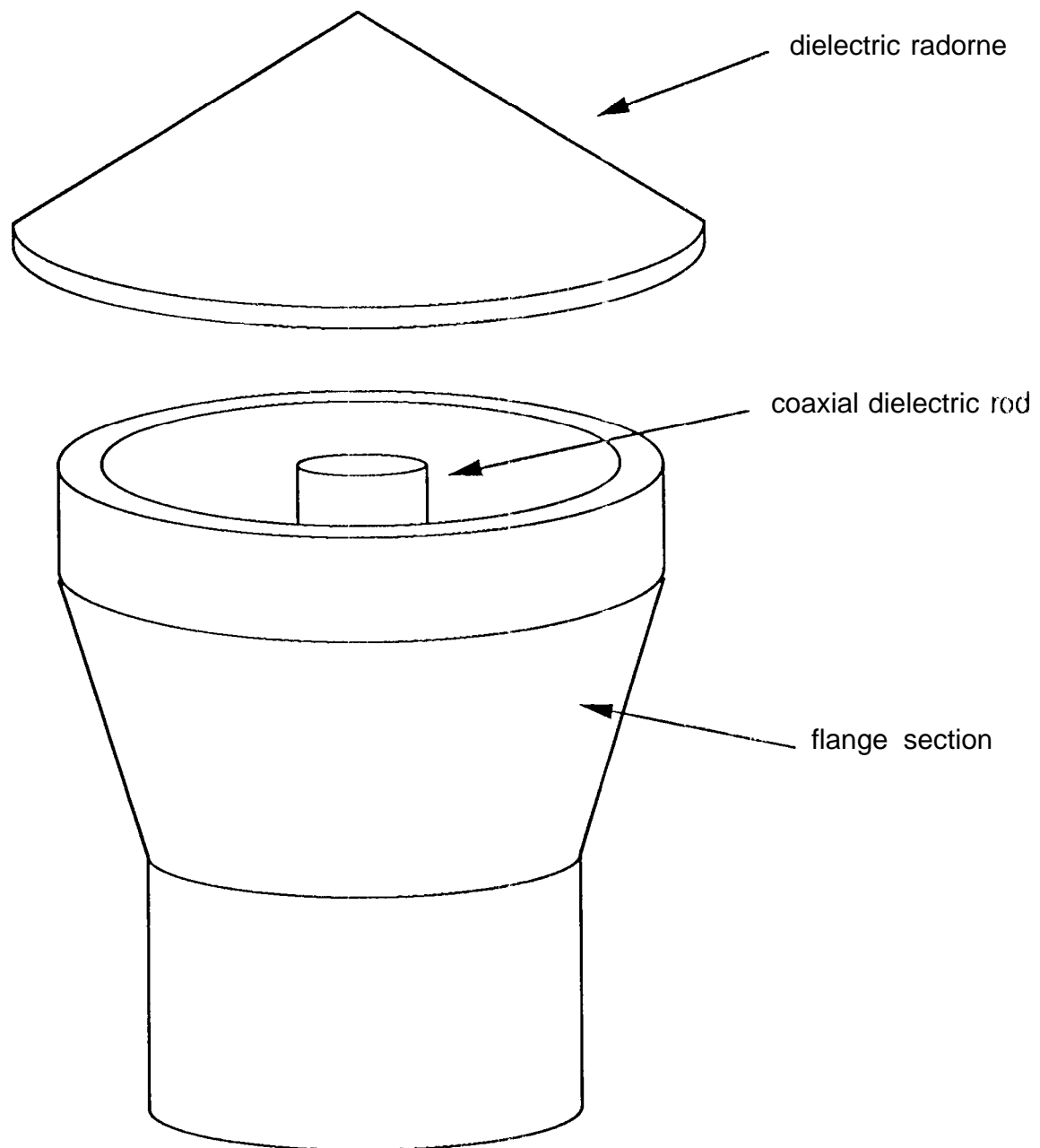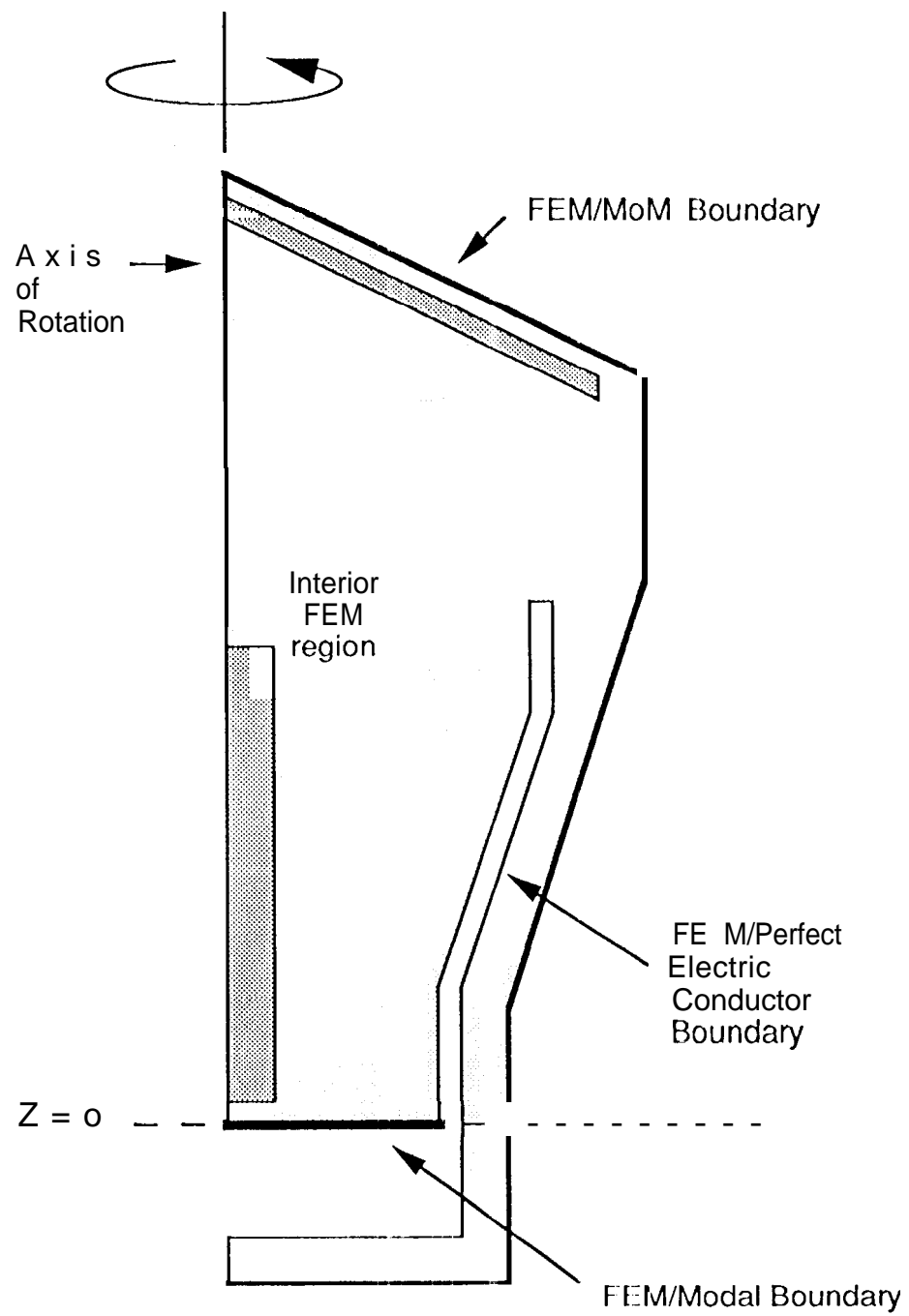
Figure 2. Computational domain for horn shown in Fig. 1.
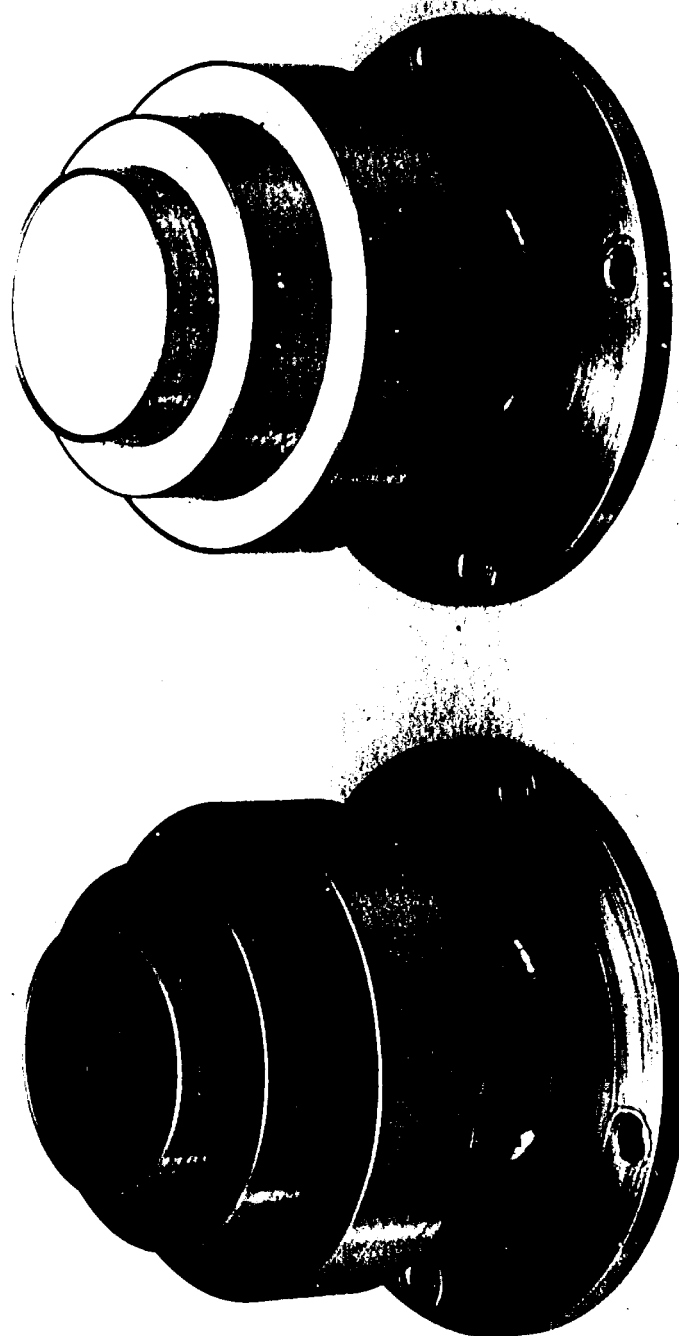
Figure 5.   Pathfinder Low Gain Antenna Horn (LGA)

Axis of Rotation
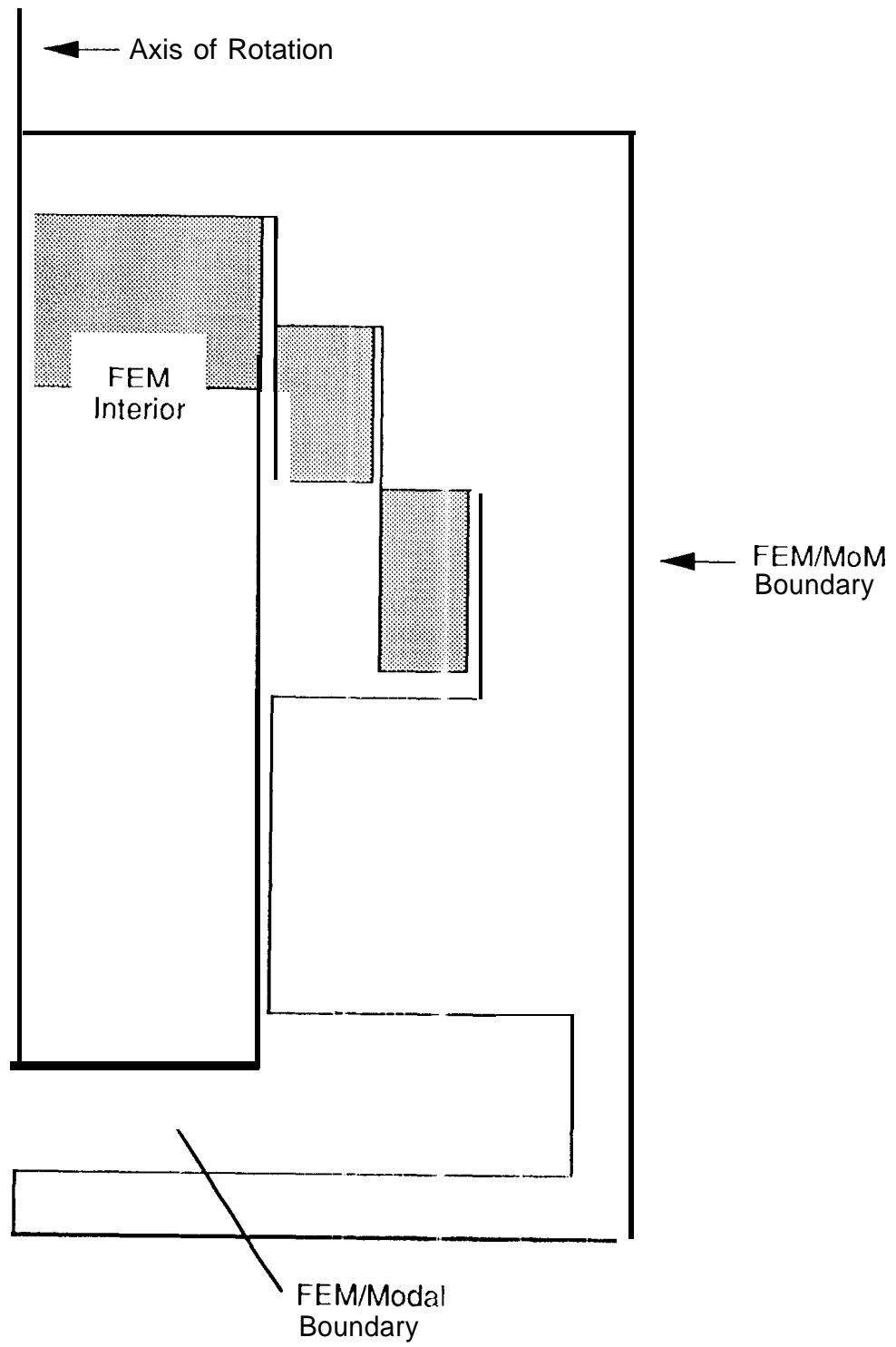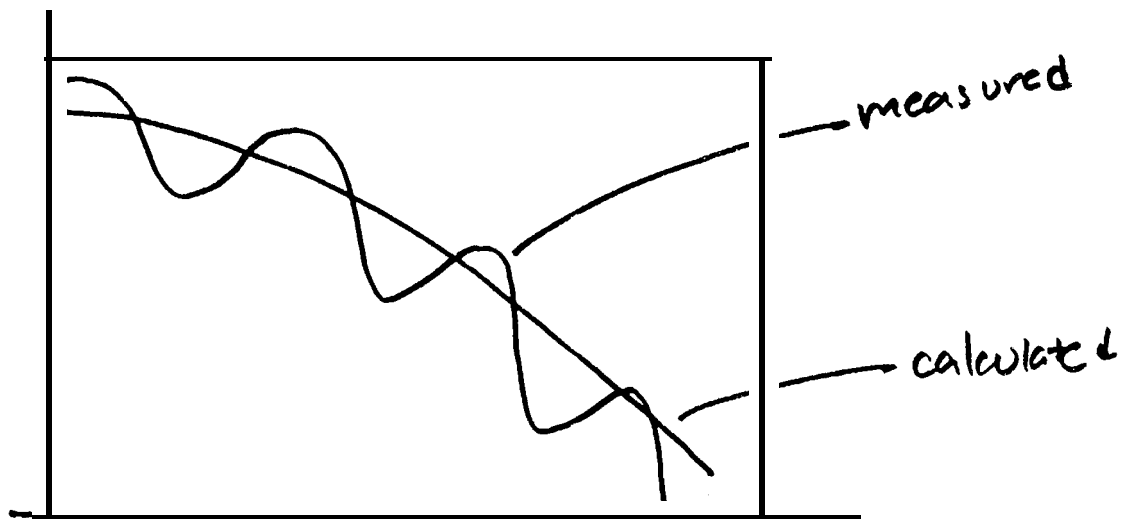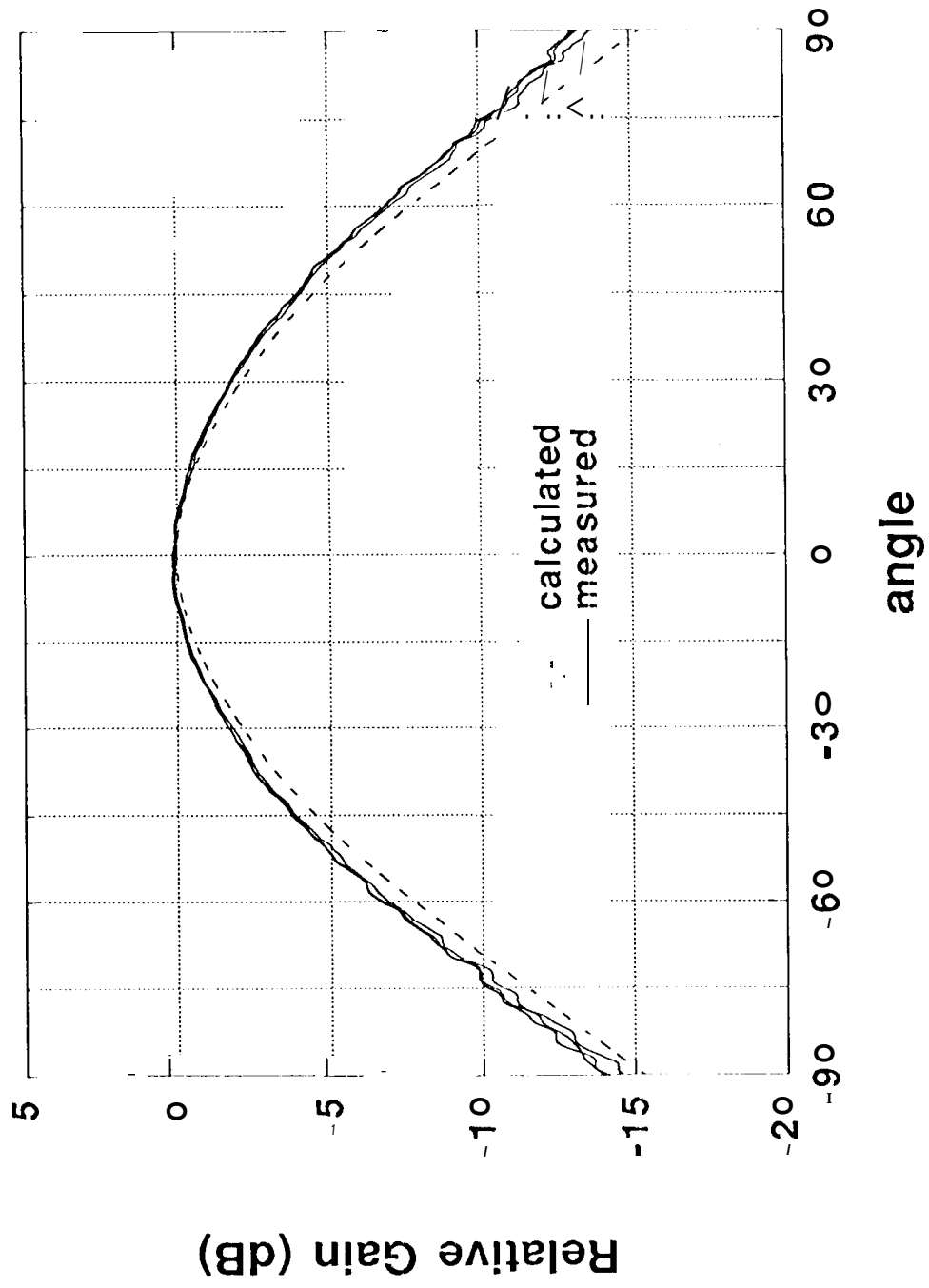
FEM
Interior

FEM/MoM
Boundary

FEM/Modal
Boundary

Figure 5. – "comparison of measured & calculated reflection coefficient for Pathfinder LGA horn" is still being prepared.
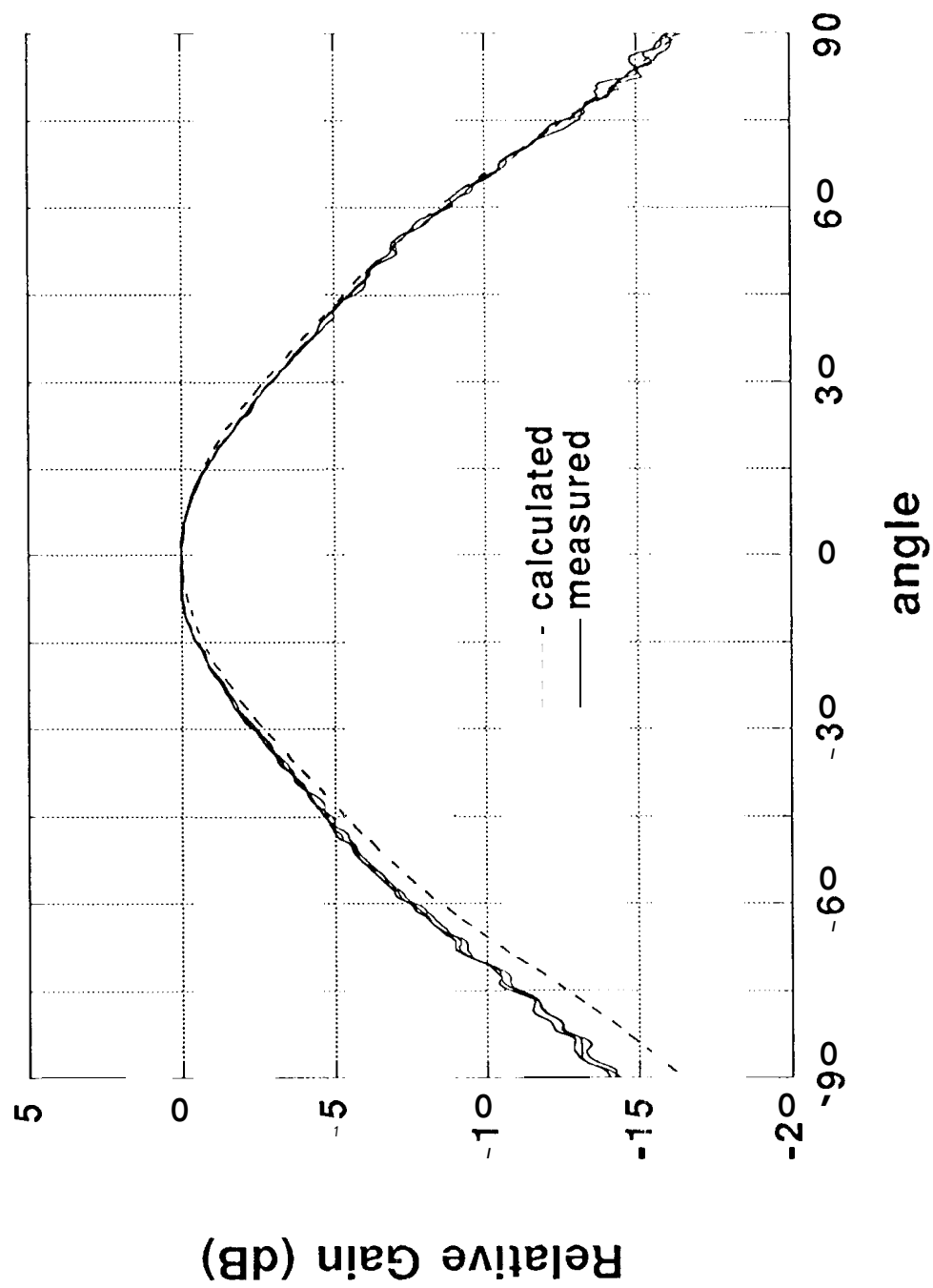

(see body of paper for explaination of ripple in measured pattern)

Pathfinder LGA far-field pattern: co-pol at f = 7.175 GHz
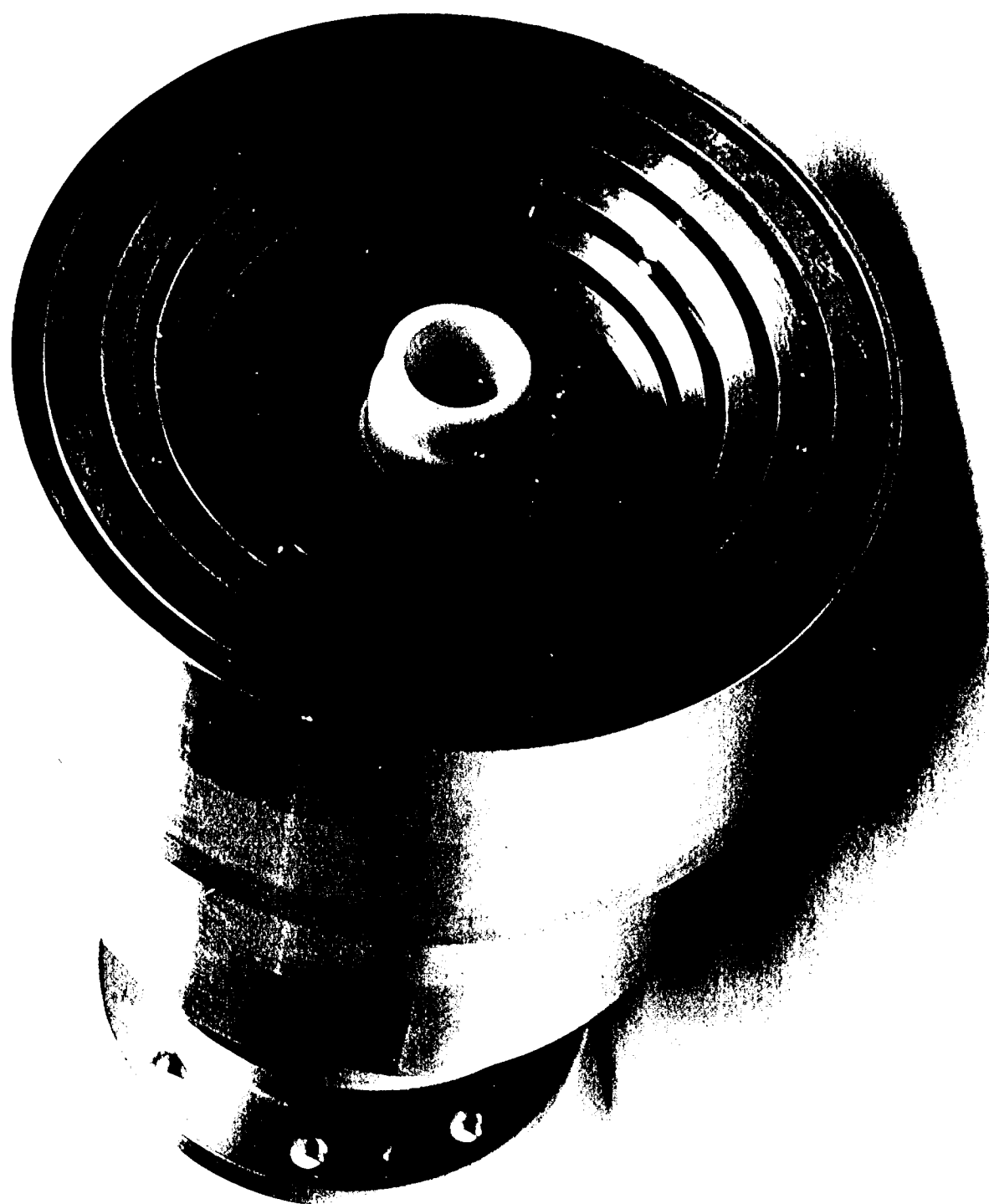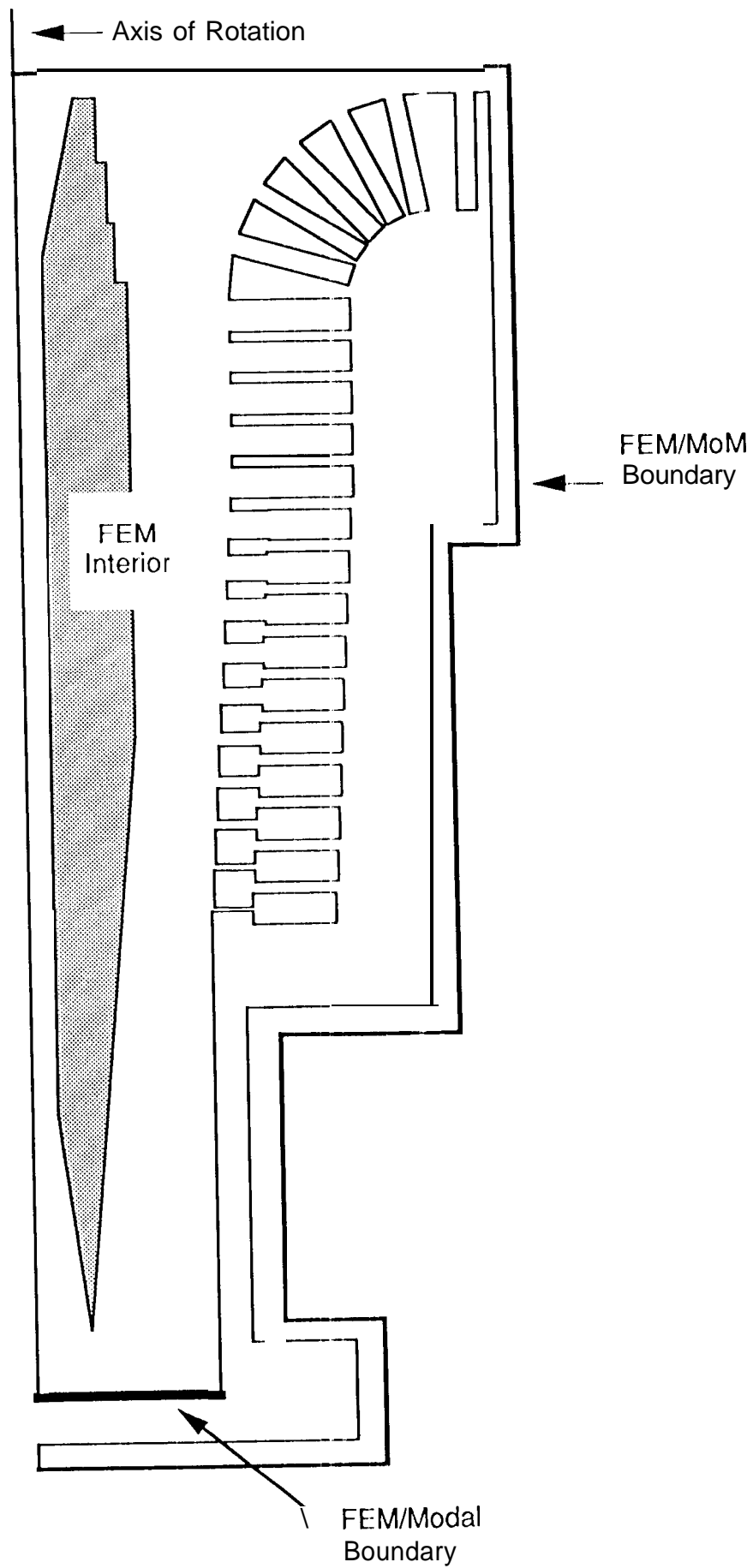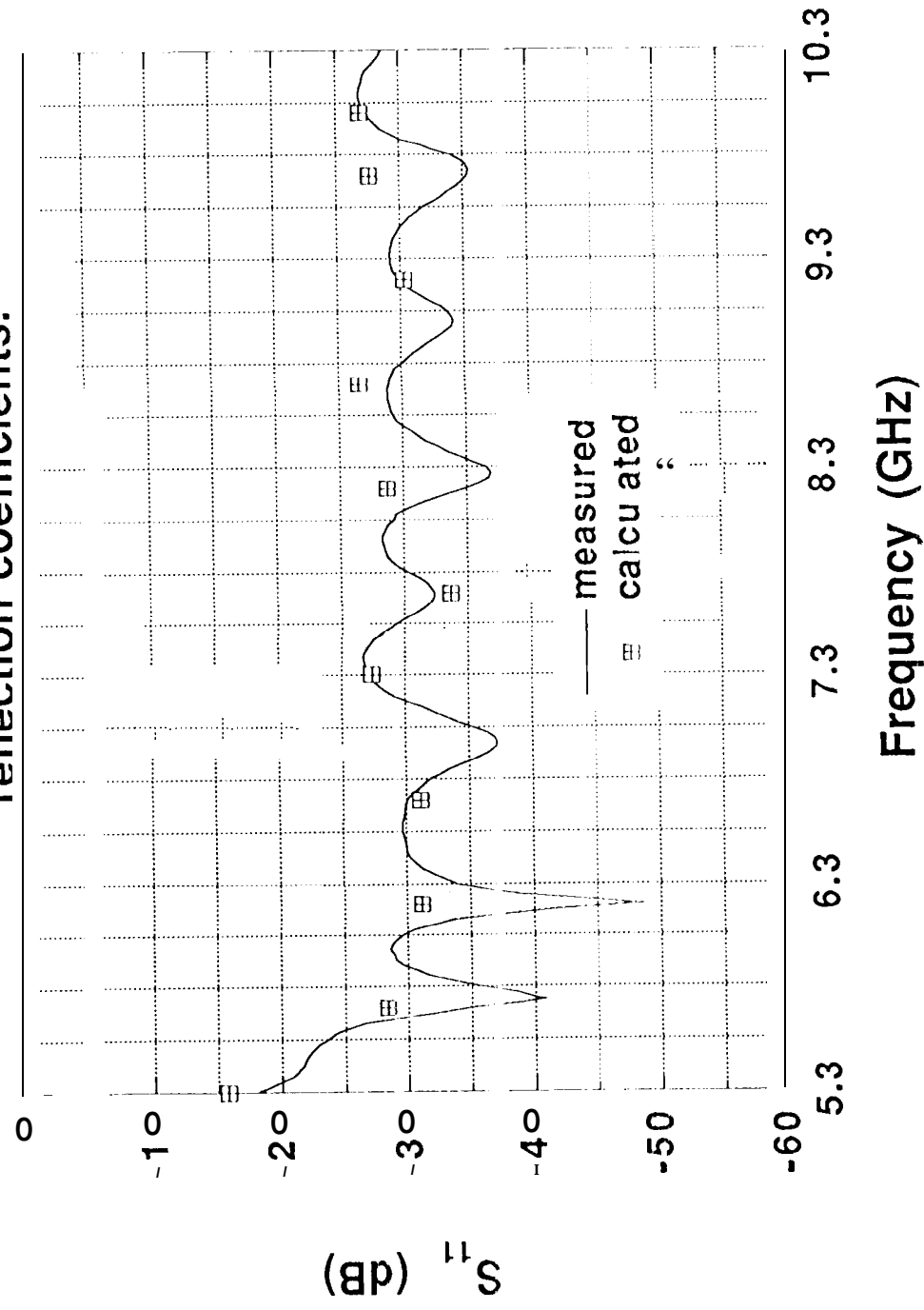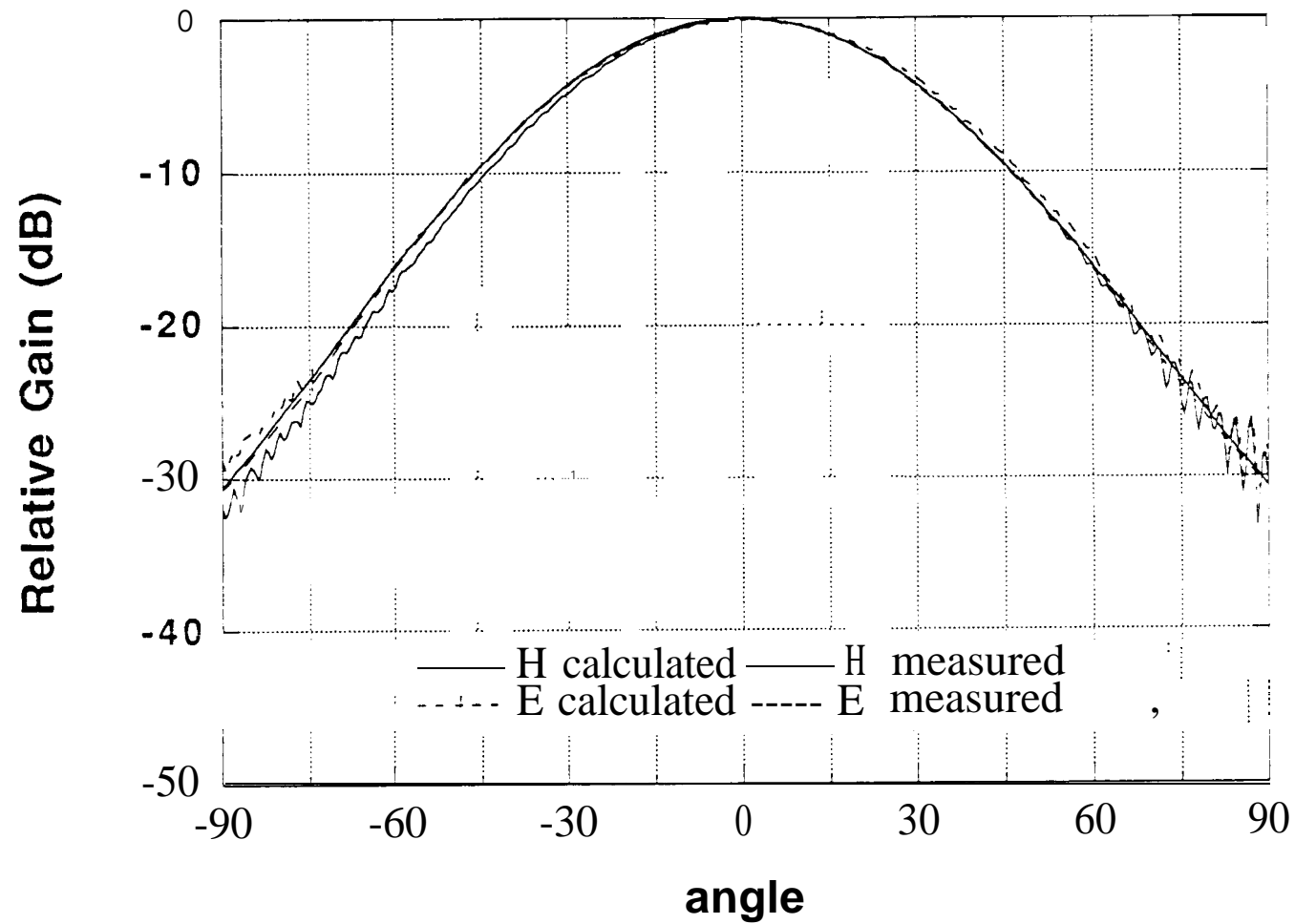
Pathfinder LGA far-field pattern:
co-pol at f = 8.425 GHz

- - calculated
—— measured

angle

Relative Gain (dB)

Figure 8. Prime Focus Feed Horn with SWE.

Axis of Rotation

FEM Interior

FEM/MoM Boundary

FEM/Modal Boundary

PFF horn:
comparison of measured and calculated
reflection coefficients.

PFF horn:
far-field patterns at f ▫8.2 GHz

PFF horn without SWE:
far-field patterns at f = 8.2 GHz